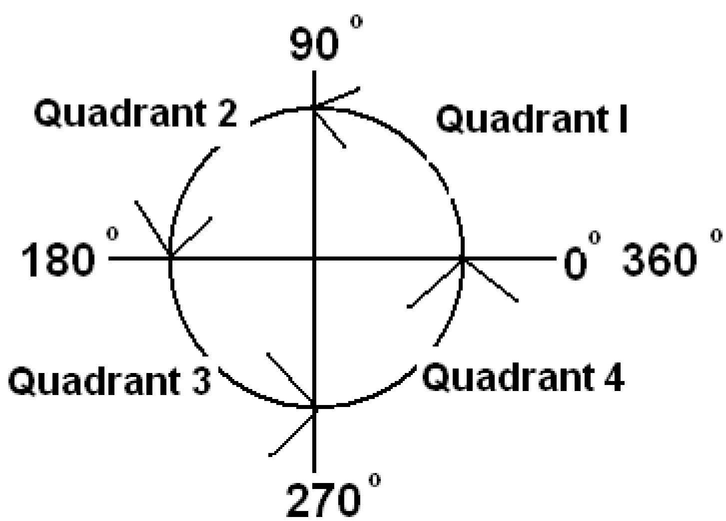
converting between degrees and radians.

There are 360 degrees in a circle. That number is pretty much arbitrary but it was probably chosen because it breaks down into whole numbers in critical places. For example, a straight angle is exactly 180 degrees and a right angle is exactly 90 degrees.

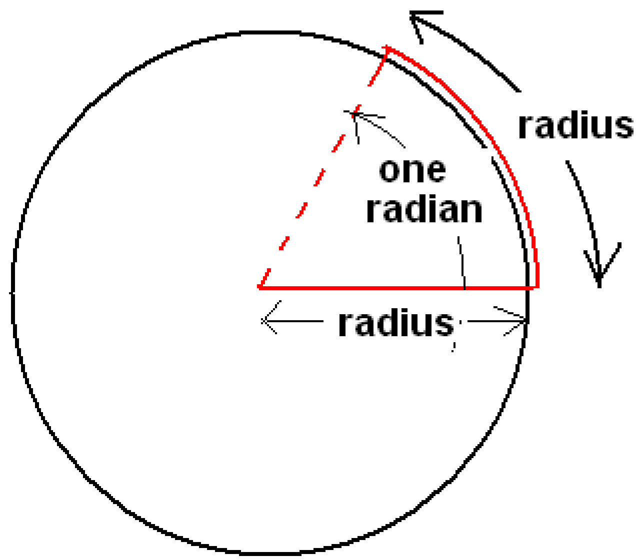
There are other angles that you will find are useful, like 30 degrees, 45 degrees and 60 degrees. These angles will be common in our discussions in trig.



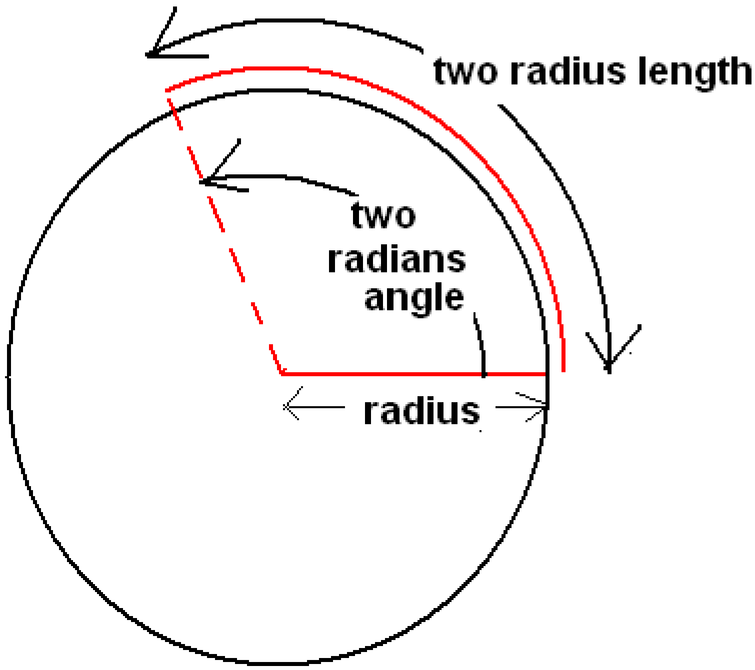
On the graph, angles are generated from the positive x axis in a counter-clockwise direction. These angles shown are the quadrantal angles that divide the plane into four quadrants I thru 4.

Radian measure is distinct from degrees in that it is not an arbitrary unit of measure for an angle. It arises naturally when a circle is drawn.

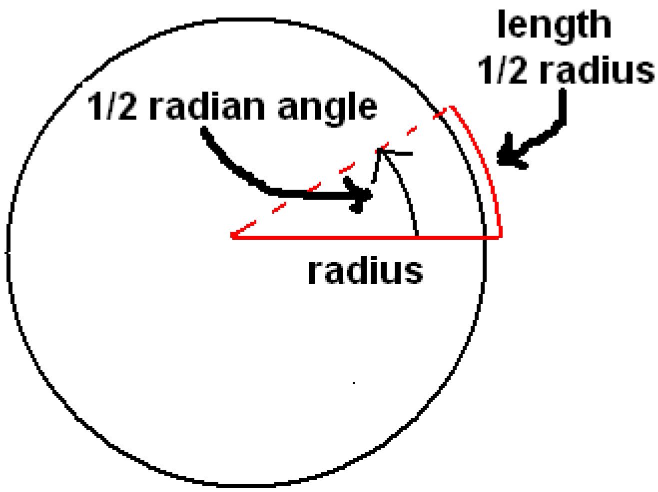
One radian is the angle formed at the origin when the radius of that circle is laid onto the circumference.



If you were to lay the radius onto the circumference twice, you'd get an angle of two radians.



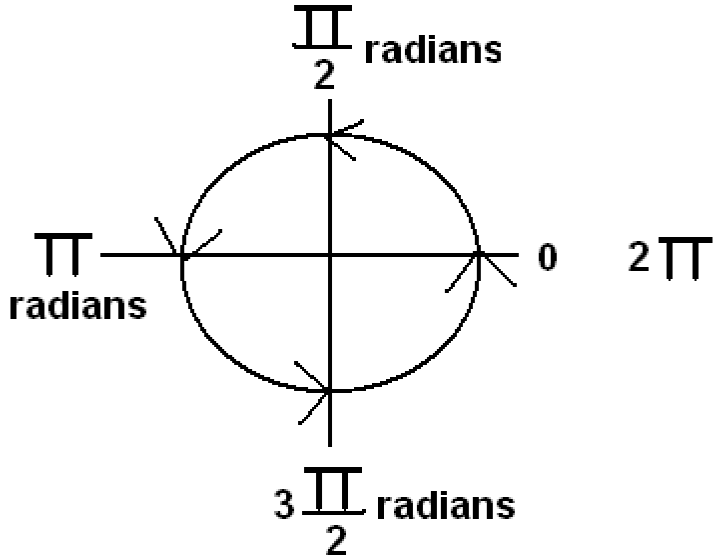
If you laid half the radius on the circumference you'd get 1/2 of a radian for the angle.



You recall the formula that calculates the entire circumference from the radius is:

Circumference = 2 X pi X radius.

This means that you can fit exactly 2 X pi radiuses (radii) onto the circumference of a circle. That means that 360 degrees is the same a 2 Pi radians because they both measure the angle for a complete circle. And if this is true, it means that half a circle (or 180 degrees) is equal to half 2 Pi or simply pi. And since 90 degrees is half 180 degrees, in radians half Pi must be 1/4 of a circle. So 2 pi radians is a whole circle, pi radians is 180 degrees (half a circle) and pi/2 is 90 degrees (1/4 of a circle).



Now the question is how do we convert angles in degrees into angles in radians and vice versa? The answer is to create a simple conversion constant. We know that Pi radians is equal to 180 degrees. So we then know that the two conversion constants would be (180 degrees)/(pi radians) and (pi radians)/(180 degrees) depending on which way we wanted to convert.

Let's say we wanted to convert 45 degrees into radians. We would choose the constant so that the degree units would cancel out but the radian unit would remain.

the angle in radians = (45 degrees) X (Pi radians)/(180 degrees)
The degree units cancel out and leave (45 X Pi)/180 radians.
This boils down to pi/4 radians. Which is correct.

Note that I did not change pi to 3.1416.. in order to get a numerical answer. If you keep the pi, you can imagine the angle in radians better than you can if you just have a number. This is because we know that 2pi is a whole circle, pi is half a circle and pi/2 is ninety degrees. We now know that 45 degrees is half of

that or pi/4.

Converting from radians to degrees is a matter of multiplying how many radiasn you have by the conversion factor in the form (180 degrees) / (pi radians) that way the radian units will cancel out and leave you with degrees.

When you use a calculator, the answers will be in numbers. Some calculators leave the pi in the answer. Of course in the end, in physical problems in science and engineering you need an actual number, pi won't do. So you tell the calculator to give you the numerical answer.

The unit circle.

If your circle is a unit circle, i.e. it has a radius of one, then the length of the arc is exactly the same as the angle in radians. So if you laid 1.5 radiuses (radii) on the circumference and your radius was one, you would have an angle of 1.5 radians. When you have a unit circle, the arc length in radiuses is the same as the angle in radians.

However, suppose the circle was inflated until its radius was two or three. You can see if you laid one of these longer radii on the circumference that the arc would be longer. (twiced as long if the radius is two and three times as long if the radius is three. Yet the angle would still be one radian.

This is the formula that relates the radius of a circle, the arc length and the angle in radii (aka radians).

arc length = (radius) X (angle in radians)

This formula says if you are given the radius of a circle and the angle in radians, you can calculate the length of the arc on the circumference that is cut off by that angle. You should keep your units of measure in mind to work this kind of problem.