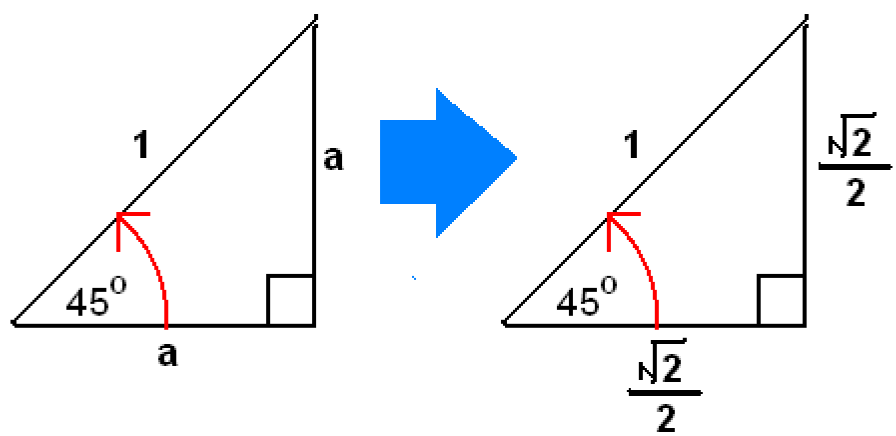
There are three angles in radian measure, in the first quadrant, that are used regularly in teaching mathematics.They are
pi / 6 (30 degrees).
pi / 4 (45 degrees) and
pi / 3 (60 degrees).

These angles are commonly used in examples because their functions are easy to demonstrate using Π and √2 and √3 instead of long decimals. This makes it a lot easier for manual calculations. And that, in turn, makes it easier to illustrate some of the useful properties of the Unit Circle.

Let's begin with a right triangle whose acute angles are 45 degrees (or Π / 4 rads) and whose hypotenuse is 1.



$$a^2 + a^2 = 1^2$$
$$2a^2 = 1^2$$
$$a^2 = \frac{1^2}{2}$$

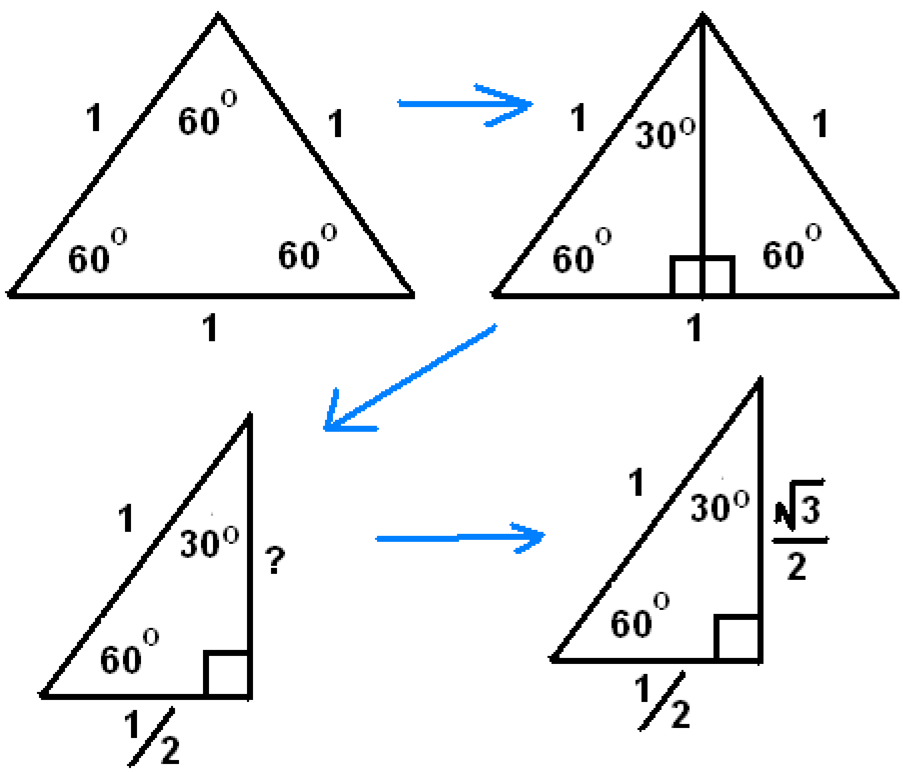
$$a = \sqrt{\frac{1^2}{2}}$$
$$a = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Using the pythagorean theorem we calculate the two other sides which gives us √2 / 2 for each side after rationalizing the result. Now we can calculate the sine and cosine functions for 45º.

sin (45º) = √2 / 2 or
sin (Π / 4 rads) = √2 / 2
and the
cos(45º) = √2 / 2 or
cos(Π / 4 rads) = √2 / 2.

Now let's do the same thing for the angles 60º and 30º

Begin by making an equilateral triangle (equal sides) whose angles would be 60º. Now divide that triangle down the middle and you get two right triangles. Each has a 60º and a 30º angle. Discard the right triangle and let's use the left one to figure out the sines and cosines for 60º and 30º.



Again, using the Pythagorean theorem we calculate the ? side to be √3 / 2.
Now we have what we need to calculate the sines and cosines of 60º and 30º (also known as Π/3 rads and Π/6 rads respectively.

sin (60º) = √3 / 2 or
sin (Π / 3 rads) = √3 / 2

cos(60º) = 1 / 2
cos(Π / 3 rads) = 1 / 2

and

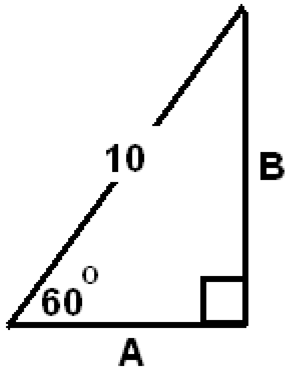
sin (30º) = 1 / 2 or
sin (Π / 6 rads) = 1 / 2

cos(30º) = √3 / 2 or
cos(Π / 6 rads) = √3 / 2

So, at this point we know the sines and cosines for the angles 30º , 45º , and 60º also known as
Π / 6 rads, Π / 4 rads and Π / 3 rads.

Now let's solve a couple of triangles (just to try these things out)

Suppose you are given a right triangle with a sixty degree angle and a hypotenuse of 10. Without using a calculator what will be the length of the other two sides (you are going to revere this piece of effete arcane technical terminology with which you can amaze your little sister at how much you have learned in college) The other two sides are called cathetus or together catheti. It sounds the way it looks.



$$\cos(60^\circ) = \frac{A}{10}$$

$$\frac{1}{2} = \frac{A}{10}$$

$$\boxed{A = 5}$$

$$\sin(60^\circ) = \frac{B}{10}$$

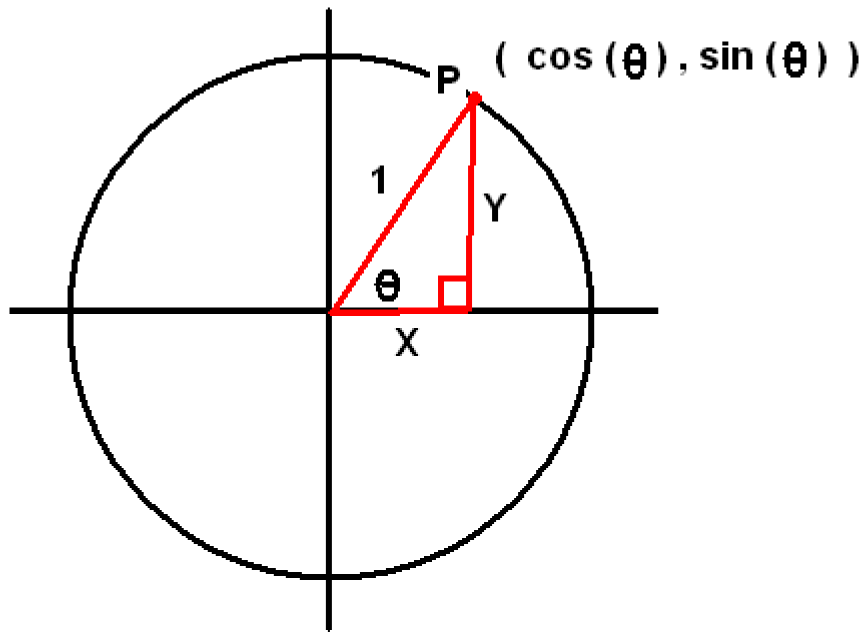
$$\frac{\sqrt{3}}{2} = \frac{B}{10}$$

$$B = \frac{10\sqrt{3}}{2}$$

$$\boxed{B = 5\sqrt{3}}$$

Taking advantage of the fact that we have committed the sin and cos of 60 degrees to memory, we can substitute the actual values into them in the above equations and solve them. The advantage of this is that we do not have to use a calculator (provided we don't mind small radicals in the answer.

Now let's look at these same angles when they are on a graph using the Unit Circle. Recall that the Unit Circle is a circle of one unit in radius centered at the origin and the angles are generated by rotating the radius counter clockwise around the origin.

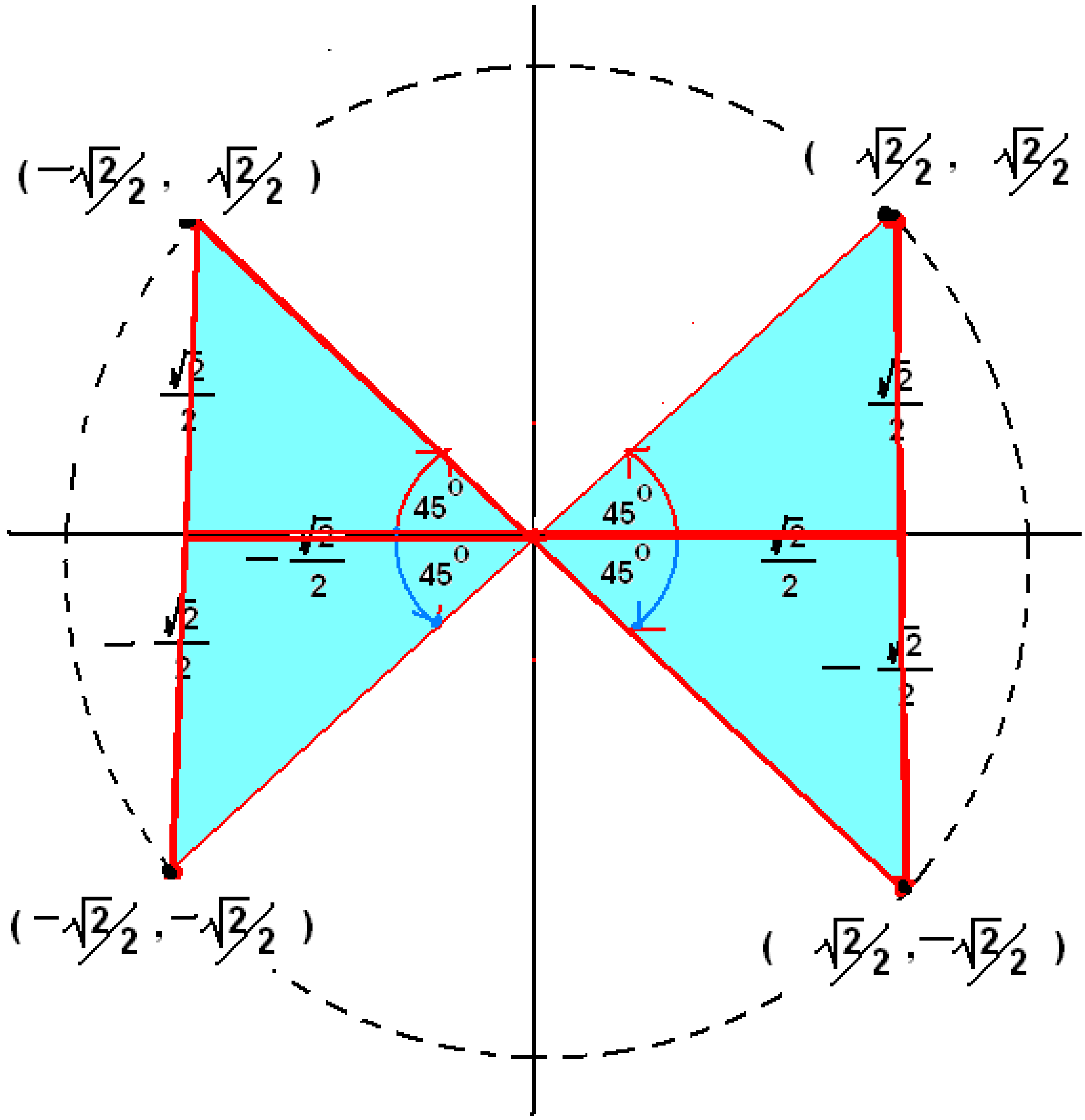


If you have a radius of 1, that is always going to be the hypotenuse of the triangle. So when you figure out what the sin is for any theta you will see that it is Y / 1 or simply Y. That means when you have a triangle with a hypotenuse of 1, the height (y) will simply be the sin (theta).

Also, if you calculate the cos (theta) you will see that it is X / 1 which is simply X. So the X side (or adjacent side) is equal to the cos (theta).

Now, since we know this, we know that for any angle (theta) in the first quadrant of a Unit Circle, the coordinates of that point are given by (cos(Θ), sin(Θ)).

So let's see what the coordinates of the points would be for a reference angle Θ = 45 degrees in all four quadrants.

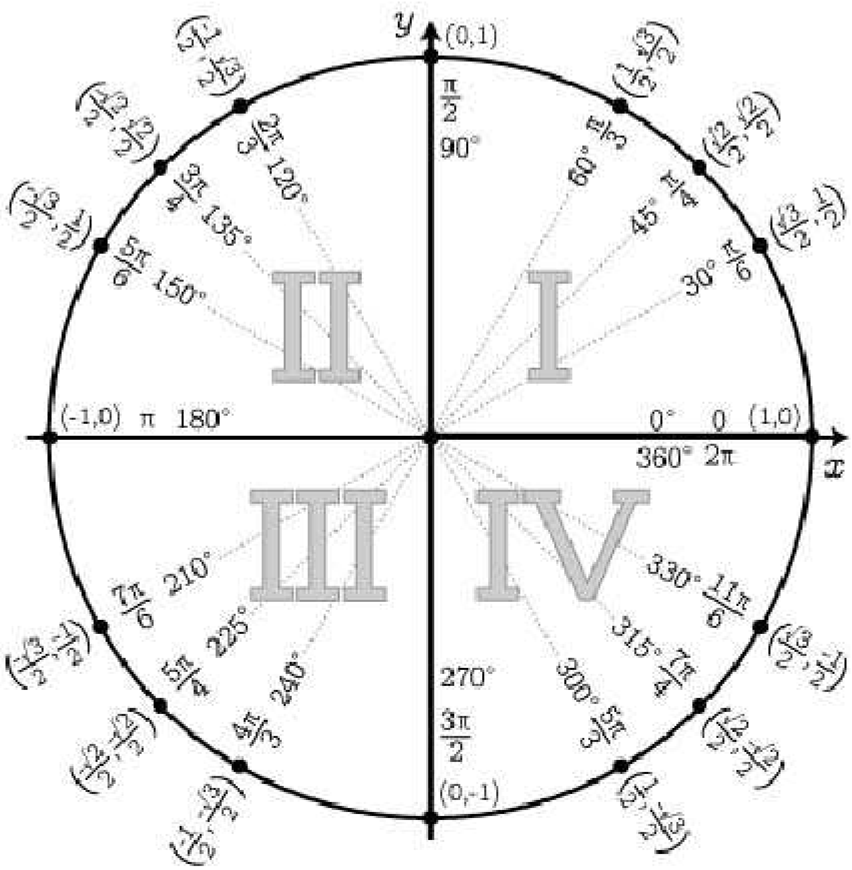


As we discussed in class, the only difference between the sin and cos of theta in the different quadrants are the signs. Otherwise, the refernce angles all have the same numerical values. That's because the ratios of the sides are the same.

To see what happens when we draw the reference angles 60 degrees and 30 degrees in all four quadrants, check out this web site. It will give you all three angles in both degrees and radians along with their coordinates.

http://www.touchtrigonometry.org/

For the time being, ignore the curves on the right in this touchtrigonometry web site, and concentrate on the information near the unit circle in the image below.



Hope this helps. Any comments you have will be appreciated.